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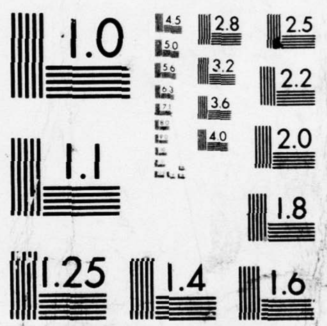

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GENERALIZATION OF THE HYPERSONIC AREA RULE

by

M. D. Ladyzhenskiy

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GENERALIZATION OF THE HYPERSONIC AREA RULE

By: M. D. Ladyzhenskiy

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К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α ɑ	Nu	Ν	ν
Beta	Β	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	Ο	ο
Delta	Δ	δ	Pi	Π	π
Epsilon	Ε	ε ϵ	Rho	Ρ	ρ ϱ
Zeta	Ζ	ζ	Sigma	Σ	σ ς
Eta	Η	η	Tau	Τ	τ
Theta	Θ	θ ϑ	Upsilon	Υ	υ
Iota	Ι	ι	Phi	Φ	φ ϕ
Kappa	Κ	ϰ κ	Chi	Χ	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	Μ	μ	Omega	Ω	ω

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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GENERALIZATION OF THE HYPERSONIC AREA RULE

M. D. Ladyzhenskiy (Moscow)

We give a generalization of the hypersonic area rule [1], proved under the assumption [2] that the entire mass of the gas is concentrated behind the shock wave during flow past thin blunt bodies, to the case when the body emerges beyond the limits of the region bounded by the surface of the shock wave, i.e., condition (3) from [1] is violated.

The flow problem reduces to the equivalent problem of nonsteady-state motion of a gas displaced by a piston. Let at some moment of time $t = t^*$ the surface of the piston come into contact with a shock wave. When $t < t^*$ the equations which describe the flow past a blunt body having resistance to bluntness X_0 and a law of the

change in cross-sectional area S [in the equivalent problem S(t) is the area of the piston] have the form [1] (for the case of axisymmetric flow see [2])

$$\frac{1}{2}\rho_\infty R^2\dot{R}=\int_0^t (p-p_\infty)\,R\,dt \tag{1}$$

$$\frac{\pi}{2}\rho_\infty R^2\dot{R}^2+\frac{p}{\varkappa-1}(\pi R^2-S)=X_0+\frac{\pi p_\infty R^2}{\varkappa-1}+\int_0^t p\dot{S}dt \tag{2}$$

Here R(t) is the radius of the shock wave which, when t < t', has axial symmetry [1] relative to the leading point of the body O (Fig. 1); p is the pressure in the region between the shock wave and the body, constant over the entire region because of the assumption of infinite compression in the shock wave; p_∞ and ρ_∞ are the pressure and density, respectively, of the unperturbed gas; $\varkappa$ is the adiabatic exponent; the raised dot indicates differentiation with respect to t.

Let us find the solution when t > t'. In accordance with the assumption of the concentration of the entire mass of the gas behind the shock wave, on those sections of the contour that project beyond the limits R - a circle (Fig. 1), the compressed layer contacts the surface of the body and the pressure is defined by Newton's formula[1].

FOOTNOTE: [1] Newton's formula with consideration of centrifugal forces

during flow past an arbitrary surface has been derived in [3,4]. END FOOTNOTE

Required: to determine the pressure in regions D_1 and D_2, constant at each moment of time; to find the laws of motion of a shock wave (on the sections where the wave does not touch the body) and the points of intersection of circle R with the body surface P_1, P_2, Q_1, Q_2.

Let us examine the solution for region D_1 (the solution is constructed analogously for D_2).

Let the equation of the body contour in polar coordinates ρ, ψ with center at point O have the form $\rho = \rho(\psi, t)$. From the equation $\rho = R(t)$ we find the relationship for angle P_1OP_2, designated below by φ:

$$\varphi \equiv \psi_2 - \psi_1 = \Phi[R(t), t] \tag{3}$$

where $\Phi(R, t)$ is a known function; ψ_1 and ψ_2 are the angular coordinates of points P_1 and P_2, respectively.

The impulse equation written for region D_1 coincides with

equation (1). To derive the energy equation let us present a picture of the flow at two close moments of time t and t + dt. In Fig. 2 the contour of the body and the shock wave are depicted at moment of time t by solid lines, and at moment t + dt - by the dashed line. Let us isolate on circle R at moment t small elements of the compressed layer S_1P_1 and S_2P_2 which occur on the body at time t + dt, such that $S_1P_1 \longrightarrow S'_1P'_1$ and $S_2P_2 \longrightarrow S'_2P'_2$. Striking the body surface, the mass of gas contained in S_1P_1 and S_2P_2 loses, in accordance with concepts of Newton's theory, the normal velocity component relative to the body. As a result, at points P_1 and P_2 the body is acted on by concentrated forces whose expressions are written below (7).

The kinetic and thermal energy of the mass of gas comprising an element of the compressed layer $S'_1P'_1$ is equal to the sum of the kinetic energy of element S_1P_1 and the energy imparted to the gas due to the action of concentrated force at the point of intersection of circle R with the body contour. The energy of element $S'_2P'_2$ is determined analogously.

The energy conservation equation for the remainder of the gas in region D has the form

$$\frac{\rho_\infty\varphi}{4}\frac{d}{dt}(R^2\dot{R}^2)+\frac{1}{\varkappa-1}\frac{d}{dt}(S_Dp)=\frac{p_\infty\varphi R\dot{R}}{\varkappa-1}+p\dot{S}_R \qquad (4)$$

The first term on the left in (4) expresses the change in kinetic energy of the gas mass lying on arc S_1S_2; the second term is the change in internal energy in region D; S_D designates the area included between the shock wave and the body.

The first term on the right in (4) corresponds to the increment of energy in region D due to internal energy of the unperturbed medium, while the second is due to the work of the piston.

The term $\dot{S}_R$ designates the time derivative of the area swept by the arc of body contour P_1P_2 within circle R. We can write the following equalities:

$$\dot{S}_D = \varphi R\dot{R} - \dot{S}_R, \qquad \dot{S}_R = \int_{P_1}^{P_2} N(s)\, ds \tag{5}$$

where the integral in the expression for $\dot{S}_R$ is taken along the arc of body contour s from point P_1 to point P_2; N is the normal rate of movement of the contour, which is a given function of s.

Let us write an expression for the concentrated force q acting at the point of intersection of circle R with the body contour. We have $q = \mu V_n$, where μ is the mass reaching the body surface per unit of time; V_n is the component of gas velocity, normal to the body surface, relative to the body. We can write the following equalities:

$$V_n = N - \dot{R}\cos\chi, \qquad \mu = \frac{mV_n}{\sin\chi}, \qquad m = \frac{\rho_\infty R}{2} \tag{6}$$

Here χ is the angle between the normals to the body contour and

to the shock; m is the linear density of the mass in the compressed layer behind the shock wave (we assume N > R cos χ).

Finally, for the concentrated forces at points P_1 and P_2 we get

$$q_1 = \frac{\rho_\infty R}{2\sin\chi_1}(N_1 - \dot{R}\cos\chi_1)^2, \qquad q_2 = \frac{\rho_\infty R}{2\sin\chi_2}(N_2 - \dot{R}\cos\chi_2)^2 \tag{7}$$

Thus, the complete system of equations when t < t' is written in the form of (1), (2), while when t > t' it is written in the form of (1), (3)-(5). The solutions to both systems when t = t' are "sewn together" as follows:

$$R(t'-0) = R(t'+0), \qquad \dot{R}(t'-0) = \dot{R}(t'+0), \qquad p(t'-0) = p(t'+0) \tag{8}$$

The first and third equations of (8) are obvious; the second condition expresses the absence of concentrated forces applied to the shock wave.

Now let us formulate a generalized area rule with hypersonic flow past thin blunt bodies.

For two blunt bodies with equal values of the blunting resistance, having identical parts projecting beyond the lmits of circle R (these can be called the "Newtcn edges") and identical laws

of change of the areas included within circle B, the laws of a change in pressure and motion of shock waves coincide in regions where the compressed layer does not contact the body. The surface of the shock wave in these regions retains its axial symmetry.

From this it follows that the drag forces acting on the bodies are equal, since these forces are comprised of three components:

1 - forces acting on the "Newton edges";

2 - forces acting on the bodies in regions D_1 and D_2;

3 - concentrated forces defined by formula (7).

It is essential to note that the contours of the cross sections of comparable bodies should not have angular points in those sections which project beyond the limits of circle B, or at points P_1, P_2, Q_1, Q_2.

In conclusion I would like to express my thanks to M. N. Kogan for his very useful discussions.

Received 18 January 1961

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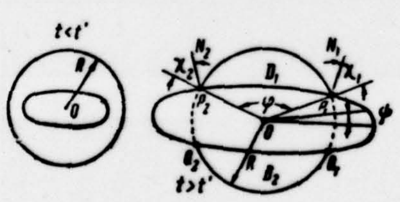

Fig. 1.

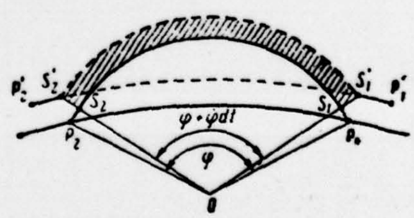

Fig. 2.